APPENDIX B: CHARACTER TABLE SUMMARY

classes	Γ_1	Γ_2	.....	Γ_s	Γ_{s+1}	.....	Γ_m
size	**1**	**1**	**.....**	**1**	$\boldsymbol{h_{s+1}}$	**.....**	$\boldsymbol{h_n}$
$\boldsymbol{\chi_1}$	1	1	.....	1	1	.....	1
$\boldsymbol{\chi_2}$	1	roots of unity					
......	.....						
$\boldsymbol{\chi_r}$	1						
$\boldsymbol{\chi_{r+1}}$	n_{r+1}	sums of roots of unity					
.....	.....						
$\boldsymbol{\chi_m}$	n_m						
Order	**1**	$\boldsymbol{k_2}$	**.....**	$\boldsymbol{k_s}$	$\boldsymbol{k_{s+1}}$	**.....**	$\boldsymbol{k_n}$

(1) m = # conjugacy classes = # irreducible characters
(2) n_i = deg χ_i (always divides |G|)
(3) r = |G/G′| = # linear characters
(4) s = |Z(G)| = # classes of size 1
(5) $\Sigma\, n_i^2$ = |G| [follows from column orthonormality]
(6) |G| = $\Sigma\, h_i$ [class equation] h_i = |G|/|centraliser| and so divides |G|

(7) (Column orthonormality)
Distinct columns are orthogonal.
Sum of squares of the moduli down column j is $\dfrac{|G|}{h_j}$.

(8) (Row orthonormality)
Distinct rows are orthogonal when weighted by the h_j.
Sum of squares along a row (weighted by the h_j) = |G|.

(9) Columns corresponding to inverses are conjugate.

(10) The conjugate of an irreducible character is an irreducible character.

(11) If $\Gamma_j = \Gamma_j^{-1}$ then each $\chi_i(\Gamma_j)$ is real.

(12) The linear characters form a group, under multiplication, isomorphic to |G/H|.

(13) The number of linear irreducible characters = r.

(14) ker ρ_i (where ρ_i is a representation for which χ_i is the character) is the union of all conjugacy classes Γ for which $\chi_i(\Gamma) = n_i$.

(15) Every normal subgroup is one of these kernels or the intersection of two or more of these kernels.

(16) Every irreducible character χ of G/H induces an irreducible character of G.
Each conjugacy class Γ in G/H corresponds to one or more classes of G. These classes map to $\chi(\Gamma)$.

(17) $\chi_i(\Gamma_j)$ is a sum of n_i k_j'th roots of unity.
In particular, if f $k_i = 2$ then $\chi_i(\Gamma_j)$ is: ± 1 if $n_i = 1$;
0 or ± 2 if $n_i = 2$;
± 1 or ± 3 if $n_i = 3$.

(18) If π is a permutation character $\pi(\Gamma_j)$ is the number of symbols fixed by each element of Γ_j. It is usually reducible.
(19) Inducing up from subgroup H to G: value = **index** of H in G × **proportion** of class in H × **average** value of character for those elements in H.
(20) Every character is an integer sum of irreducibles. If $\chi = \Sigma c_i\chi_i$ then $\langle\chi \mid \chi\rangle = \frac{1}{\lvert G\rvert}\sum_j \lvert\chi(\Gamma_j)\rvert^2 = \Sigma c_i^2$. and $\langle\chi \mid \chi_i\rangle = \frac{1}{\lvert G\rvert}\sum_j \chi(\Gamma_j)\overline{\chi_i(\Gamma_j)} = c_i$. [In particular if $\langle\chi \mid \chi\rangle = 1$, χ is irreducible and if $\langle\chi \mid \chi\rangle = 2$, χ is the sum of two distinct irreducible characters.]